MRI: Assignment #2

Due November 9, 2021

MRI Sensitivity

In the institute, there are two MRI scanners for humans: a 3T Siemens scanner situated next to the Neurobiology building, and a 7T Siemens scanner housed in Lubin, next to the Life Science & Chemistry Library. The highest field NMR spectrometer is the DRX800 (housed right in front of Ziskind's back entrance). Complete the following table, intended to give you a rough feel for the orders of magnitude involved in typical magnetic resonance experiments:

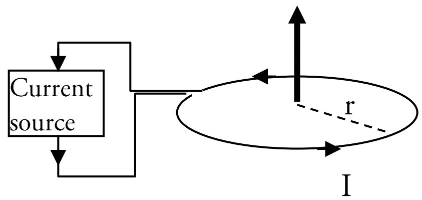
	Siemens 3T MRI	DRX800 NMR spectrometer	Earth's magnetic field
Precession frequency of proton ^{1}H nuclear magnetic moment		800 MHz	
Precession frequency of phosphorous ^{31}P nuclear magnetic moment			
Precession frequency of carbon ^{13}C nuclear magnetic moment			
Precession frequency of electron's magnetic moment			
Field strength (in Tesla)	3		
Field strength (in Gauss)			

Use external resources (eg introductory science books, Wikipedia, the lecture notes) to determine quantities of interest such as the Earth's magnetic field, the electron's gyromagnetic ratio and so forth (be careful of factors of 2π!).

Current & Fields

In this problem we'll see just how hard it is to produce the enormous magnetic fields (>1 Tesla) used in NMR. The easiest way to produce a magnetic field is by establishing a current, I, through a loop of wire. Using Ampere's law, which you might have encountered in an electricity & magnetism course, one can compute the magnetic field at the center of the loop to be:

$$|\mathbf{B}| = \frac{\mu_0 I}{2r}$$



where I is the current (in amperes), r is the wire's radius (in meters), and $\mu_0 = 4\pi\times10^{-7}$ (in Tesla × meters/Amperes).

1. Suppose you took a copper wire, made a loop out of it (of radius 0.5 meters, typical of human MRI machines) and plugged it into the power supply at your home, which is capable of delivering a max. of 16 Amperes before blowing a fuse. What would be the field at the center of the loop? What would the current I_{HF} needed to create a 3 Tesla high field?
2. A second problem which arises with copper wires is their resistance, which leads to heating. Suppose you had a current source powerful enough to generate the current I_{HF} you've found in part (a). Compute the power dissipated in the coil in Joules per second. Assume you've used a regular power cord in part (a) (i.e. the kind you use in regular household appliances). Assume it's made out of copper, and recall: (i) copper has a resistivity of $\rho = 1.7\times10^{-8}$ Ohms·meters, (ii.) the total resistance of a wire with cross section A and length l is given by $R = \rho l/A$, and (iii.) the power dissipated in a resistor R with current I flowing through it is $P=I^2R$.
3. Calculate the number of degrees a liter of water would heat up by, if it were given the thermal energy dissipated during one minute in the coil in part *(b)*.

These calculations go to show that regular conductors make for poor MRI magnets. Fortunately, the low fields and the high power dissipation can both be circumvented by using super-conductive magnets. These allow us to use very high currents in very thin wires (since there is little to no power dissipation in superconductors). Moreover, they use multiple turns of the wire to increase the field considerably. That is how the large main fields in modern MRI magnets are generated.

Magnetic Moment of the Earth

The magnetic field lines of the Earth are, to a first approximation, similar to those of a magnetic moment; i.e., it is as if there was a point magnetic moment at the center of the Earth giving off the Earth's magnetic field. **Estimate** the size of the moment (in J/T; an order-of-magnitude estimate would be fine), assuming the value of the Earth's magnetic field is 0.5 Gauss at the equator, and that the magnetic and geographical north coincide.

Bloch Equations Induce Rotations

We've seen in class that the Bloch equations in a constant field **B**,

$$\frac{d\boldsymbol{m}}{dt} = \boldsymbol{m}(t) \times \gamma \boldsymbol{B},$$

describe the left-handed precession of a spin about **B** with an angular velocity $\omega=\gamma B$. Prove this by "brute force", by showing that the time dependent expression for **m**,

$$\boldsymbol{m}(t) = R_{\boldsymbol{n}}(\theta)\boldsymbol{m}(\boldsymbol{t}),$$

satisfies the Bloch equations, where $R_n(\theta)$ is a left handed rotation matrix around an axis parallel to **B**. Guidance:

1. Assume, without loss of generality, that **B** is parallel to the z-axis. Why can you assume that?
2. What is the form of a left-handed rotation matrix about the z-axis?
3. Write down the vector expression for **m**(t) and differentiate it (to differentiate a vector, you need to differentiate each of its components).
4. Calculate explicitly the right hand side of the Bloch equations and show that you get the same result.