# MRI Primer: Assignment #6

Due Dec. 21, 2021

## Choosing the Optimal Echo Time for Functional Imaging

As we will discuss in the upcoming lecture, the oxygen-binding heme group in blood cells is diamagnetic when it's bound to oxygen, but paramagnetic when it is not bound. Without going too much into detail, this means that blood creates greater $B_0$ inhomogeneity around it when it is not bound to oxygen.

1. Will $T_2$' be longer or shorter if the blood in the voxel is oxygenated (as opposed to non-oxygenated)? Why? What about $T_2$*? Why?

Suppose you wanted to carry out a functional MRI experiment based on this contrast, which is called Blood Oxygen Level Dependent (BOLD) contrast. For example you could show some visual stimulus to the person inside the scanner and image their visual cortex with a sequence that is sensitive to $T_2$*.

2. Explain why you'd expect $T_2$* to increase in the visual cortex during the external visual stimulation.

Assume $T_2$* changed in the visual cortex from 50 ms at rest to 55 ms during activation, and that you were carrying out a simple GRE experiment.

3. Draw the difference in signal in the voxel between the activated and non-activated cases as a function of TE, the time between excitation and the formation of the gradient echo (you can use any plotting software).
4. Calculate the optimal echo time, TE, which maximizes this. Hint: it would be easier to first maximize the function plotted in (1) if you treat $T_2$* as a parameter, setting $T_2$*=$T_{2r}$ during rest and $T_2$*=$T_{2a}$ during activation, and then taking the derivative.
5. What **percent change** would you expect at this maximal enhancement? Hint: it's not big … This goes to show how small the changes are in fMRI-BOLD experiments, and how careful one should be in removing artifacts and other sources of error in the imaging process when analyzing the data.
6. Given that the SNR in the images is 100, what would be BOLD's contrast-to-noise in the voxel?

## Estimating $T_2$'

Assume your main field $B_0$ is inhomogeneous. This leads to a distribution of offsets and, as seen in class, to $T_2$' decay.

1. Suppose the spread of offsets is on the order of 100 Hz; that is, the spread of the distribution of offsets within the voxel is $\Delta\left(\gamma B_0(\boldsymbol{r})\right) \sim 100$ Hz. What would be the associated $T_2$'? Give a qualitative, "order of magnitude" answer and explain it. **Hint**: how long will it take the spins to "fan out" in the transverse plane to a point their signals interfere destructively, given that the **difference** in their precession frequencies is on the order of 100 Hz?

2. Explain why, as the voxel becomes smaller, $T_2$' becomes longer (so $T_2$' becomes less of an issue). Could you eliminate the transverse $T_2$* relaxation of the signal from the spins completely by taking smaller and smaller voxels?
3. Imagine you were a carpenter who had to produce a table 1 meter wide, to within ±1 mm accuracy; you would then say that the relative accuracy required of you was $10^{-3}$, which is quite high. The typical $T_2$ (not $T_2$'!) of water in tissue in-vivo at 3T is about 50 ms. How good must be the relative accuracy of $B_0$ within a voxel for $T_2$' to be negligible? This goes to show that the engineering demands on MRI magnets are quite extraordinary!

## Chemical Shift Artifact

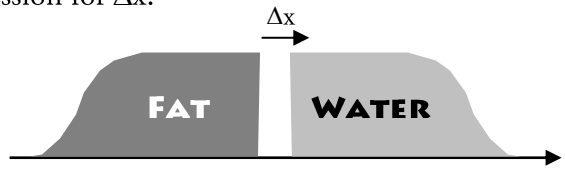
In the absence of any gradient, spins can have an intrinsic non-zero offset called their **chemical shift**. The chemical shift will differ between protons in different molecules, so if the protons (H) of water ($H_2O$) have zero offset in the rotating frame – meaning the effective field they see is zero – most protons of fat molecules will have a non-zero offset of about $\Delta\nu = 450\ Hz$ at 3T (the shift is proportional to the field). This means water and fat molecules will see different effective fields, given by:

$$\mathbf{B}_{eff}^{(H_2O)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{B}_{eff}^{(fat)} = \begin{pmatrix} 0 \\ 0 \\ \frac{\Delta\nu}{\gamma} \end{pmatrix}, \quad \Delta\nu = 450 \text{ Hz @ 3T}.$$

(in the rotating frame in which water is on resonance)

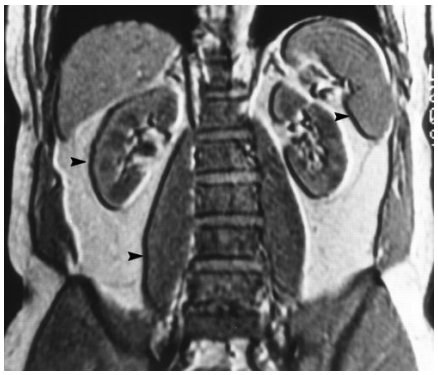
Suppose you're imaging a 1D object that has both water and fat at 3T using simple frequency encoding:


1. Explain why the fat image will be shifted by an amount Δx relative to its actual position, and derive an expression for Δx:




   Hint: With the frequency encoding gradient turned on, a water proton will resonate $\nu_{water}(z) = \gamma Gz$ as a function of position, while a fat proton will resonate at $\nu_{fat}(z) = \Delta\nu + \gamma Gz$.
2. What would your suggestion be for minimizing this artifact – that is, decreasing Δx? Note: This shift can cause serious problems at boundaries between fatty and "regular" tissue; for

example, when imaging the spinal cord, which is sheathed in fat. Another example is this coronal $T_1$-weighted abdominal image showing the lipid (bright) and water (darker) tissue that are visually separated, creating dark stripes as indicated by the arrows:



3. Looking at the above $T_1$-weighted abdominal image we notice something interesting: the chemical shift artifact appears to only happen in the left-to-right (LR) direction and not in the foot-to-head (FH) direction. The above image was acquired with standard 2D encoding, with frequency encoding applied along the LR axis and phase encoding along the FH axis. Explain why the chemical shift artifact is not seen along phase encoded axis[1].

[1] Assume phase encoding is carried out the "regular" way: the encoding time is fixed while the phase encoding gradient's strength is varied between successive scans. Yes, in case you're wondering, the fixed time is a crucial component of the reason why the chemical shift displacement is not observed in the PE direction.