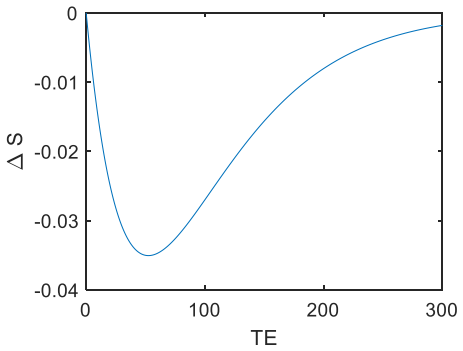
MRI Primer: Assignment #6 Solution

Choosing the Optimal Echo Time for Functional Imaging

1. T_2' will be longer: oxygenated blood is diamagnetic and creates smaller distortions in B_0 around it, meaning the signal will decay slower. T_2* will also get longer because T_2 remains unchanged while T_2' gets longer.
2. The difference in signal, also proportional to the contrast, is

$$\Delta S = S_0\left(\exp\left(-\frac{TE}{T^*_{2,act}}\right) - \exp\left(-\frac{TE}{T^*_{2,rest}}\right)\right)$$

Plot for S_0=1:



(the overall shape remains the same even if the amplitude of the signal, S_0, changes)

3. Visually you can see the minimum in the plot occurs at around TE=52 ms. This is confirmed by taking the derivative of ΔS with respect to TE and equating it to zero, and solving for TE:

$$\frac{d(\Delta S)}{dTE} = S_0\left(-\frac{1}{T^*_{2,act}}\exp\left(-\frac{TE}{T^*_{2,act}}\right) + \frac{1}{T^*_{2,rest}}\exp\left(-\frac{TE}{T^*_{2,rest}}\right)\right) = 0$$

This can be solved numerically to yield the optimal TE=52.4 ms.

4. The % signal change is simply $\frac{\Delta S}{S_{rest}} = \frac{0.033}{0.35} \approx 10\%$, irrespective of S_0.
5. If the SNR is 100, then $\frac{S_0}{\sigma} = 100$ where sigma is the standard deviation of the noise, and CNR $= \frac{\Delta S}{\sigma} = 100 \cdot 0.033 \approx 3.3\%$.

Estimating T_2'

1. If the spread of frequencies in the voxel is about 100 Hz, then it will take about $\frac{1}{100\ Hz}$ for the spins to "fan out" in the xy-plane, which is about 10 ms. This is also the corresponding T_2'.

2. As the voxel becomes smaller the macroscopic variation in B_0 in the voxel (e.g. due to susceptibility artifacts or magnet imperfections) becomes smaller as well, leading to a smaller $\Delta(\gamma B_0(\boldsymbol{r}))$ and a longer $T_2' \sim \frac{1}{\Delta(\gamma B_0(\boldsymbol{r}))}$. T_2^* could not be eliminated completely for two reasons: First, T_2^* also contains a contribution from T_2 which will never go away. Second, some static inhomogeneities in B_0 occur on a mesoscopic scale – e.g., due to iron depositions inside the cell, or the presence of a vein with deoxygenated blood – and our voxels are macroscopic, so making them 0.5 mm instead of a 1 mm will not change the mesoscopic B_0 inhomogeneities.
3. If B_0=3T the correspond Larmor frequency is approximately 123 MHz. For T_2' to be negligible is must be much smaller than T_2, so we demand $T_2' \ll T_2 \sim 50\ ms$, which becomes a demand on the spread of frequencies in the voxel: $\gamma \Delta B_0 \sim \frac{1}{T_2'} \ll \frac{1}{50\ ms} \sim 20$ Hz. Therefore, we require a relative accuracy of $\frac{20\ \text{Hz}}{123\ \text{MHz}} \approx 10^{-7}$.

Chemical Shift Artifact

1. Frequency encoding simply adds a gradient and acquires data and does a Fourier transform. This gives you a distribution of intensities at different **frequencies** (only by assigning a frequency to each position in the sample you can assign positions to frequencies). The presence of a constant offset for lipids means its frequencies in the final Fourier transform will be shifted by a certain amount relative to a water spin at the same position. This amount is $\Delta x = \frac{\Delta \nu}{\gamma G}$. To see this, note that the frequency of a spin at position z for water is $\nu_{water}(z) = \gamma G z$ and for fat is $\nu_{fat}(z) = \Delta\nu + \gamma G z$. This means the lipid spin will resonate at a frequency that is $\Delta\nu$ higher than a water molecule at the same position – i.e. it will be shifted in the Fourier transform by an amount $\Delta\nu$. When you convert frequency to position via $\nu = \gamma G z$ it will appear $\Delta\nu = \gamma G \Delta z$ farther away, or $\Delta z = \frac{\Delta \nu}{\gamma G}$.
2. Very simple: just make the readout gradient bigger! Of course this has drawbacks. For example, hardware limitations might prevent you from doing so. If G is too large and you switch it on and off too fast you also run the risk of eddy-currents (residual currents in the coils due to the fast switching of currents).
3. Phase encoding encodes position via the phase of the spins. All spins evolve for the same amount of time, T, and the gradient amplitude is changed between acquisitions. This means all spins, regardless of their offset $\Delta\nu$, get encoded with the same **relative** phases (it's true that lipids will accumulate a global extra phase $\Delta\nu_{lipids} \cdot T$, but this does not translate into an improperly encoded **position**, because the **relative** phases are what we use to encode a spin's position in phase encoding; do an example with 2nd order phase encoding to convince yourself of this!).