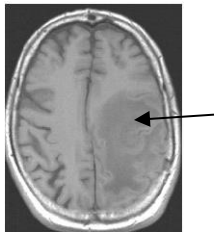
MRI Primer: Assignment #7

Due December 28th, 2021

Imaging Edema & Cancer

Edema is a pathophysiological condition in which water accumulates in the space between the cells due to pressure differences between the tissue and blood vessels. It leads to swelling and can be caused by many pathologies, from diabetes and heart failure to trauma. Edema is characterized on an MR level by longer T_1 values compared to healthy tissue. Let's see how best to use a spoiled gradient echo (spoiled GRE) to image it - that is, to make it "pop out" on a T_1-weighted image[1]. Some cerebral edema surrounding a cancerous tumor can be seen as a large dark patch on the following T_1-weighted image below:



This huge dark blob here is the edema in case you missed it.

1. Plot the signal equation for a spoiled GRE as a function of T_1 for α=90°, TR=1 sec. Use that to explain why edema appears darker than the surrounding tissue (Assume T_2, TE, M_0, etc. are the same for all cases considered unless otherwise specified[2]).
2. Suppose for simplicity T_1=1 sec for "healthy tissue" and T_1=2 sec for edema. What is the signal intensity, constast to noise and total scan time for the following four parameter choices:
    a. TR=50 ms, α=10°.
    b. TR=50 ms, α=90°.
    c. TR=1 sec, α=10°.
    d. TR=1 sec, α=90°.

    Assume we are acquiring a single slice with 256×256 spatial resolution, using frequency encoding along one of the axes and phase encoding along the other axis. Which combination gives the best SNR for edema? Which gives the worst SNR, and why? Which gives the best contrast to noise ratio? Assume some fixed noise term with unit standard deviation.
3. What does the fact that the cancerous growth is "invisible" in the above image tell you about its T_1?

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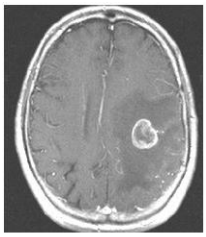
[1] Edema is actually easier to visualize on T_2 weighted images due to larger differences in T_2 and contrast formation mechanisms, but I really want to ask a question about spoiled GRE ...

[2] This is actually **not** true because T_2 and M_0 do differ between edema and healthy tissue, but let's keep things simple for the sake of this problem.

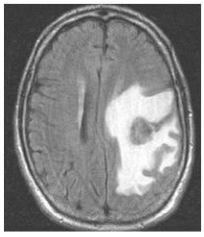
4. To better visualize the cancer, contrast is injected. The cancer breaks down and/or interferes with the blood brain barrier which allows for some contrast to leak into the growth, substantially shortening its T_1 value, yielding an image that looks like this:



T1 with contrast

Explain why the cancerous tissue which takes up the contrast agent appears very bright compared to its surroundings (also, note that the edges of the growth take up most of the contrast, owing to the fact that they are the most permeable – this is a common pattern in imaging of tumors).

5. Edema is actually usually best seen on T_2 weighted images, such as this FLAIR image (which is T_2-weighted and has an additional pulse sequence component which nulls the cerebrospinal fluid signal, making it appear dark):

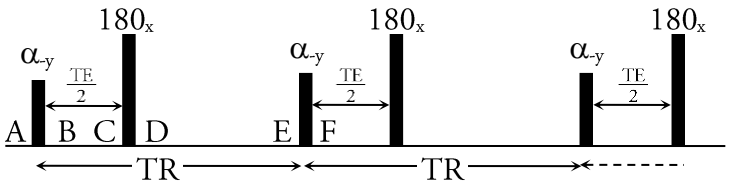


Flair

The acquisition sequence for FLAIR is similar to a spin echo acquisition. Question: is T_2 of edema shorter or longer than that of the surrounding healthy tissue?

Dynamic Equilibrium of a Spin Echo Sequence

Our derivation of the equilibrium signal for a pulse train only holds for a gradient echo sequence. The spin echo sequence is somewhat different, because it has an extra 180° pulse:



1. Derive the equilibrium signal for a spin echo sequence; i.e., calculate M_{xy}^{B}. Assume magnetization is spoiled before each successive α-pulse ($M_{xy}^{E}=0$), and that pulses have

negligible duration. **Hint**: treat M_z^A as unknown and calculate $M_z^B, M_z^C, M_z^D, M_z^E$. Use the fact that this is a spoiled sequence! Finally, use the fact that you are in dynamic equilibrium, and so $\mathbf{M}_A=\mathbf{M}_E$. Solve for M_z^A, and use that to compute M_{xy}^B, and show that:

$$M_{xy}^B = \frac{\left(1 + e^{-\frac{TR}{T_1}} - 2e^{-\frac{\left(TR-\frac{TE}{2}\right)}{T_1}}\right)}{1 + \cos(\alpha)e^{-TR/T_1}} \sin(\alpha)\, M_0$$

Note: This gives you the transverse magnetization at point B right after excitation. What you image would be the transverse magnetization a time TE/2 after the 180° pulse; this is simply given by $M_{xy}^B e^{-\frac{TE}{T_2}}$. Because this question is about T_1 contrast we confine ourselves to the magnetization at point B (adding the T_2 term is not relevant for our subsequent discussion).

2. Show that, for $TE<<T_1$, the SE dynamic equilibrium reduces to

$$M_{xy}^B \approx \frac{\left(1 - e^{-\frac{TR}{T_1}}\right)}{1 + \cos(\alpha)e^{-\frac{TR}{T_1}}} \sin(\alpha)\, M_0$$

How does this differ from the gradient echo dynamic equilibrium (which we derived in class and does not have the 180° pulses)?
By the way: $TE<<T_1$ is actually a good assumption, since TE will be chosen to be on the order of T_2 (if we want T_2 contrast) or much shorter (if we don't) and, for most tissues, $T_2<<T_1$.

3. Use your favorite plotting software to plot the signal of the sequence M_{xy}^B as a function of TR/T_1 for $\alpha = 10°, 45°, 90°, 135°, 170°$. Compare that to plots of the signal from a gradient echo experiment for the same flip angles.
4. Compare the SNR and CNR of a SE and GRE experiments for WM and GM at 3T (T_1=1.0 and 1.5 sec) with TR=0.1 sec and TR=1 sec at $\alpha = 10°$. Which sequence has the better CNR? Can you provide an intuitive explanation for your answer? (Hint: T_1 contrast is created by letting the longitudinal magnetization M_z "try" to recover back to its thermal equilibrium in between pulses; what happens to M_z during the GRE sequence at dynamic equilibrium? What happens to it during the SE sequence?)